Theoretical exposition of Single-electron Quantum Wires

Moh.Adhib.U.A, Kamsul Abraha

Department of Physics, Faculty of Mathematics and natural Sciences, gadjah Mada University

Sekip Utara PO BOX BLS.21 Yogyakarta 55281

Abstract

This paper will review phenomena in a semiconductor quantum wires which their size, shape, and interaction can be presicely controlled through the use of nanofabrication technology. The focus of this paper is to study GaAs-based quantum wire trapped one dimentional harmonic potential. Both Zeeman splitting and spin orbit interaction are neglected in our calculation. The calculation of single particle quantum wires show the energy spectrum is not degenerate and all energy levels are sparated by the same interval value. In the presence of an external magnetic field, energy spectrum single-electron quantum wires enter Landau regime when its cyclotron frequency (ω_c) is much larger than the frequency associated with the confinement potensial (ω_0) and magnetics field increase the confinement potential.

Keywords : Quantum wire, cyclotron frequency, confinement potential

*Corresponding author.

Email address : adhib_2004ugm@yahoo.com

I.Introduction

Semiconductor technology has come along in leaps and bounds since the development of the transistor in 1948. Progress in semiconductor device fabrication and carbon technology allow for the construction of several low dimensional structures at the nanoscale and therefore novel transport phenomena have been revealed. The growth of nano- technology is started with the Thouless idea[1] about possibilities to reduce the conductor dimension to the size of just few atoms.

The Thouless idea has been replied immediately by to growth of microelectronic fabrication which has succeeded in developing a controlled system which only consist of just few paticles. The particle restriction into two dimension is known a *quantum well*, the advanced confinement in one dimension is referred to as a *quantum wire,* and electron confinement in all three spatial directions is known as a *quantum dot*. This dimensional evolution[2] can be see in **fig 1.**

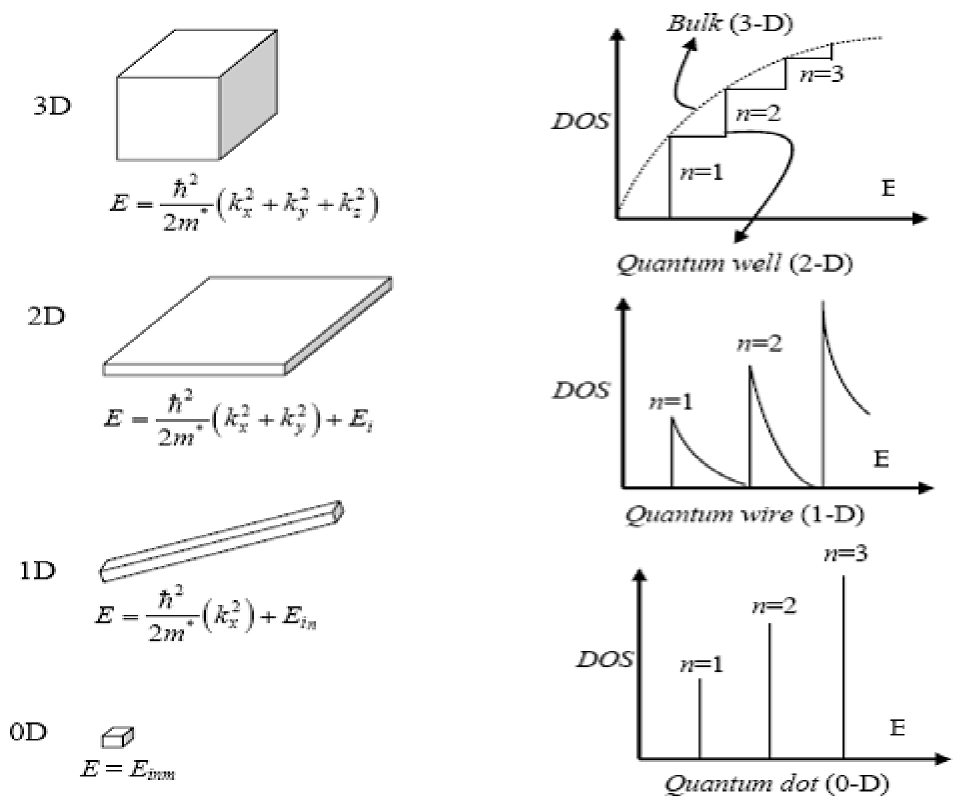


Figure1.*Evolution of spatial confinement and its corresponding density of state (DOS)*

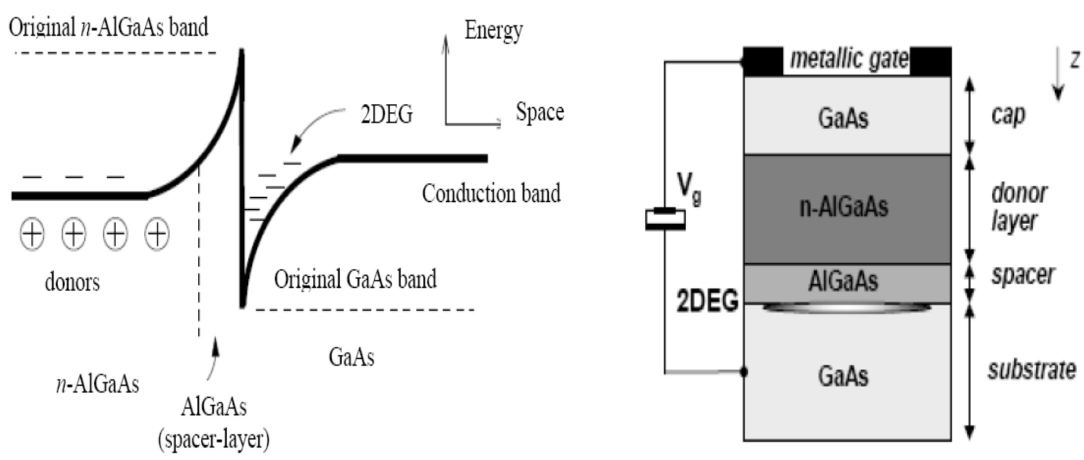
This paper is organized as follows. In **Sec.II,** we introduce a theoretical model for the single-electron quantum wires. Analytical results for the quantum wire in the presence of an external magnetic field perpendicular to the wire is discussed in **Sec.III**. Finally, **Sec.IV** gives some conclusion and further work to be done in near future.

II.Theoretical Model for a Single-electron Quantum Wires

Quantum wires are usually made by restriction a two dimensional electron gas (2DEG) in the semiconductor heterostructure[3]. Galium Arsenide (GaAs) and Alumunium Galium Arsenide ($Al_xGa_{1-x}As$) are semiconductors with similar lattice constant (e.q. 5.65 A for GaAs) and thus can be brought together to form a heterostructure. At room temperature the bandgap of GaAs is 1.424 eV, while the bandgap of AlGaAs depends on mixing factor x that can be approximated by[4]

$$E_g\,(x) = 1.424 + 1.429x - 0.14x^2\ (\text{eV}) \qquad (1)$$

with $0<x<0.44$, i.e., it varies in the range 1.424-2.026 eV, leading to discontinuity at their surface. Working with the GaAs-AlGaAs system mentioned above, a two dimensional electron system is typically created by doping the n-AlGaAs in the heterostructure (**fig 2**). At negative voltages, the 2-DEG is electrostatically squeezed from underneath the gate in to the region under the uncovered openings. As a resulth, the conduvtion band elevtron will be confined in the *x-y* plane, on addition to the vertical confinment at the heterointerface. By changing the lithography of the split-gate,one can reduce the 2-DEG to 1-D forming *quantum wires*.



a) **b)**

Figure2. a)*Two dimensional electron gas (2DEG) seen long the confining spatial dimension. b)Heterostructure AlGaAs-GaAsin the vertical cross- section.*

By dopping $Al_xGa_{1-x}As$-layer with the Si-layer as it has a lower band edge. Some of the electron added will eventually migrate into GaAs and will still be attracted by the positive donors of the conduction band discontinuity and the donor potential, they are then trapped in a narrow potential well, and the advanced confinement in one dimension is referred to as *quantum wire*.

Let us consider a 2DEG filling the *xy* plane. The charge carriers have momentum **P** = ($\hat{p}_x, \hat{p}_y$) and effective mass m*. The particle is confined along the *y* direction by the harmonic oscillator potential

$$V(y) = \frac{1}{2} m^* \omega_0^{\ 2} y^2 \tag{2}$$

where ω_0 is the oscillator frequency. The single particle Hamiltonnian is

$$\hat{H} = \frac{\hat{P}_x^{\ 2} + \hat{P}_y^{\ 2}}{2m^*} + \frac{1}{2} m^* \omega^2{}_0 \hat{y}^2 \tag{3}$$

So that the Schrödinger equation in the rectangular cooedinate system is

$$\left[\frac{\hat{P}_x^{\ 2} + \hat{P}_y^{\ 2}}{2m^*} + \frac{1}{2} m^* \omega^2{}_0 \hat{y}^2 \right] \psi(x,y) = E\psi(x,y) \tag{4}$$

where ψ (*x,y*) is the eigenfunction and *E* is the corresponding energy (eigen value). Using the variable sparation method, the wave function ψ (*x,y*) can be written as [5]

$$\psi(x,y) = \Re(x)\ \varphi(y) \tag{5}$$

with $\Re$ (*x*) is the solution for the *x* part of Schrödinger equation given by

$\Re(x) = \frac{1}{\sqrt{2\pi}} e^{ik_x x}$, with $k_x = \frac{\sqrt{2m^* E}}{\hbar}$ and the eigenvalue for this solution is E_{k_x} = $\frac{\hbar^2 k_x^{\ 2}}{2m^*}$. Moreover, the solution for the *y* part is obtained from the Schrödinger equation

$$\left[\frac{-\hbar^2}{2m^*} \left(\frac{d^2}{dy^2} \right) + \frac{1}{2} m^* \omega_0^{\ 2} y^2 \right] \varphi(y) = E\varphi(y) \tag{6}$$

which is the Schrödinger equation for one dimensional harmonic oscillator with the energy corresponding to

$$E_n = \left[n + \frac{1}{2}\right]\hbar\omega_0 \tag{7}$$

with eigenfunction is given by

$$\varphi(y) = \left(2^n n!\left(a\sqrt{\pi}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\left(\frac{y}{a}\right)^2} H_n\left(\frac{y}{a}\right)\right) \tag{8}$$

Therefore, the total solution of single-electron quantum wire is given by

$$E_{n,k} = E_n + E_{k_x} = \left[n + \frac{1}{2}\right]\hbar\omega_0 + \frac{\hbar^2 k_x{}^2}{2m^*} \tag{9}$$

and

$$\psi(x,y) = \frac{1}{\sqrt{2\pi}} e^{ik_x x}\left(2^n n!\left(a\sqrt{\pi}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\left(\frac{y}{a}\right)^2} H_n\left(\frac{y}{a}\right)\right) \tag{10}$$

with $a = \left(\frac{\hbar}{m^*\omega_0}\right)^{\frac{1}{2}}$.

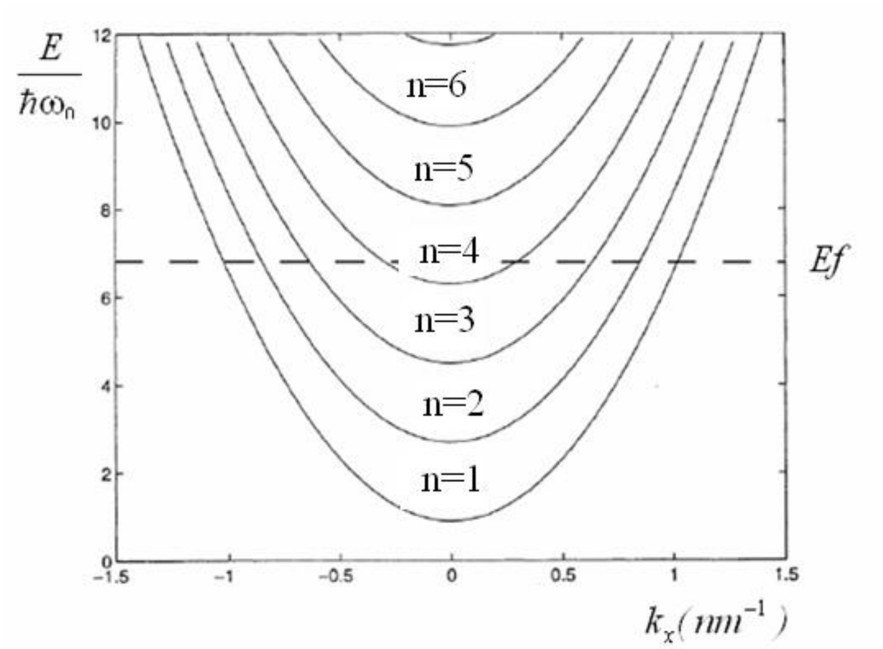


Figure3. *Energy spectrum of single-electron quantum wire*

Figure 3 shows the energy spectrum of the *quantum wire* which is not degenerate and all energy levels are spareted by the same interval value.

III.Magnetic Effect on The Single-electron Quantum Wire

Let us now consider the influence of an external magnetic field perpendicular to a 2D single-electron *quantum wire*. A magnetic field has a negligible effect on both Zeeman spin splitting, $g\mu_B\vec{B}$ which is only ~0.025 meVT^{-1} in GaAs since g_{GaAs} = -0.44[6], and the spin orbit interaction. The Hamiltonian of system is then given by

$$\hat{H} = \frac{1}{2m^*}\left[\left(\hat{p}_x + \frac{e\hat{y}B_0}{2c}\right)^2 + \hat{p}_y^{\ 2}\right] + \frac{1}{2}m^*\omega_0^{\ 2}\hat{y}^2 \tag{11}$$

so That the Schrödinger equation is

$$\left\{\frac{1}{2m^*}\left[\left(\hat{p}_x + \frac{e\hat{y}B_0}{2c}\right)^2 + \hat{p}_y^{\ 2}\right] + \frac{1}{2}m^*\omega_0^{\ 2}\hat{y}^2\right\}\psi(x,y) = E\psi(x,y) \tag{12}$$

Since Eq(12) contains the operator $\hat{y}, \hat{p}_x, \hat{p}_y$ and the wave function takes the form of wave in the *x* direction, then $\hat{p}_x\psi = \hbar k_x\psi$ and Eq(12) can be rearranged (using separation of variable) to give

$$\left[\frac{\hat{p}_y^{\ 2}}{2m^*} + \frac{1}{2}K(y - y_0)^2\right]\psi + \frac{\hbar^2 k_x^{\ 2}}{2m^*}\psi = E\psi \tag{13}$$

where

$$y_0 = \frac{-cB\hbar k_x}{2\left(e^2 B_0^{\ 2} + 2m^* c^2\omega_0^{\ 2}\right)} \tag{14}$$

and

$$K = \frac{e^2 B_0^{\ 2} + 2m^* c^2\omega_0^{\ 2}}{2c^2} \tag{15}$$

The term in square bracket of equation (12) is the Hamiltonian for one dimensional harmonic oscillator with frequency

$$\omega_c{}^2 = \frac{K}{m^*} = \frac{e^2 B_0{}^2 + 2m^* c^2 \omega_0{}^2}{2m^* c^2}$$

$$= \frac{1}{2}\omega_c{}^2 + \omega_0{}^2 \qquad (16)$$

where $\omega_c = \dfrac{eB_0}{m^* c}$ is the cyclotron frequency.Hence the eigenvalues correspond to

$$E_n = \left(n + \frac{1}{2}\right)\hbar\left(\frac{1}{2}\omega_c{}^2 + \omega_0{}^2\right)^{\frac{1}{2}} + \frac{\hbar^2 k_x{}^2}{2m^*} \qquad (17)$$

with wave function is given by

$$\psi(x,y) = A_n H_n \left\{ \sqrt{\frac{m^* \omega_0}{\hbar^2}}(y - y_0)\exp\left[\frac{1}{2}\sqrt{\frac{m^* \omega_0}{\hbar^2}}(y - y_0)\right] e^{ik_x x} \right\} \qquad (18)$$

From Eq (17) we can see that the spevtrum energy is not degenerate and all energy levels are sparated by the same interval value. The spectrum energy can be plotted in **fig 4**

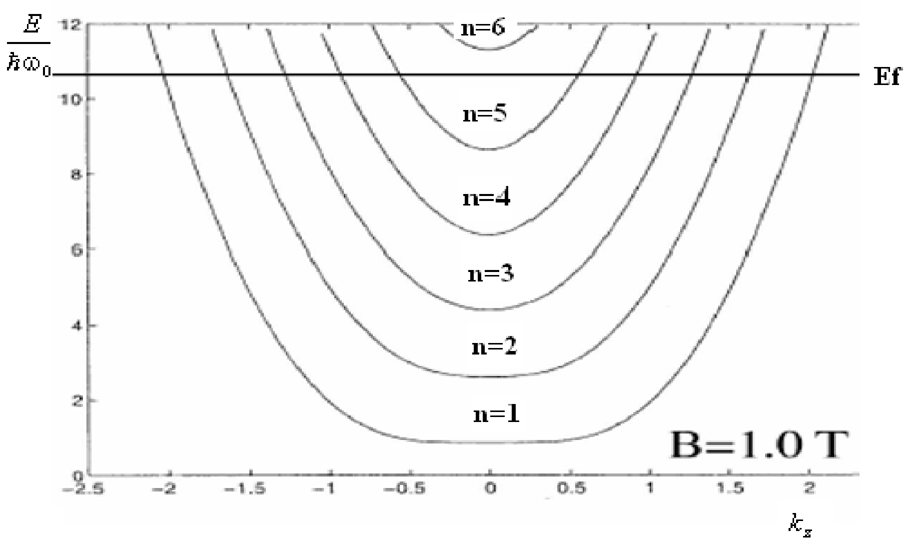


Figure 4. *Energy spectrum of single-electron Quantum wires in an external magnetic field*

In comparing figure 4 with figure 3 we can see that the external magnetic field increase the confinement potential.

It can been from Eq (16) that, when $\omega_c >> \omega_0$, electrons will enter the *Landau regime* where $E_{n,k_x} = \left[n + \frac{1}{2}\right]\hbar\omega_c + \frac{\hbar^2 k_x{}^2}{2m*}$. In this region, the harmonic potential is smaller then the external magnetic field, so the electrons inside the wire behave as they are fre particle under the magnetic field. Thus the spectrum can be seen in **fig 5** as a few lowest eigen values (in units of $\hbar\omega_0$) Vs the magnetic field *B* (expressed as ω_c / ω_0).

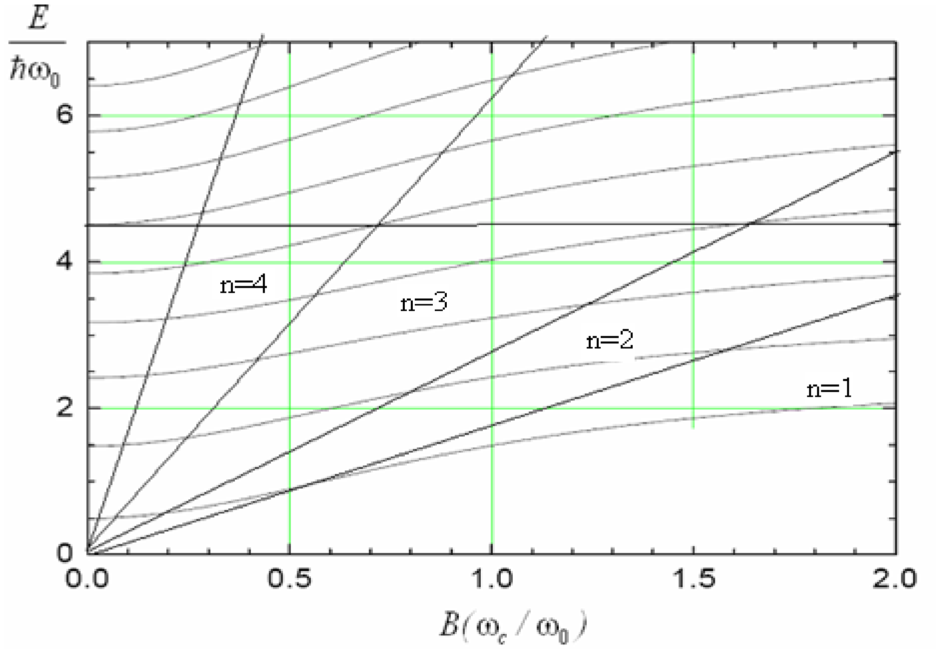


Figure 5 Plot of few lowest eigenvalues for single electron Vs magnetic field

IV. Conclusion

Analytical study of single- electron *quantum wire* has been reviewed in this work. Analytical solution shows that energy spectrum is not degenerate and all energy levels are separated by the same interval value.In the presence an external magnetic field, the energy spectrum enters the *Landau regime* when $\omega_c >> \omega_0$, and therefore increasing the confinement potential.

Some future work is still necesseery to be done. For instance, theoretical investigation of two-electron *quantum wire* using analytical approach. Moreover, the physics properties of quantum wires, e.g. their thermodynamic and statistical properties are also interesting subjects to be studied in future work.

V. Reverences

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